Cumulative Prospect Theory and Radner Theory: A Critical Assessment from Nigeria

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Authors' contributions

This work was carried out in collaboration between both authors. Author SOO designed the research topic, wrote the literature review of the manuscript. Author PEO wrote the conclusion and recommendations of the study. Both authors read and approved the final manuscript.

ABSTRACT

Aims: This study undertook a critical comparative assessment of cumulative prospect theory and Radner theory. The aim is to examine investors' behavior in the financial market using these theories. The specific objectives of the study were to examine if there are similarities between the cumulative prospect theory and Radner theory; ascertain the implications of the cumulative prospect theory to financial market; find out the implications of the Radner theory to financial market; and assess the drawbacks of the cumulative prospect and Radner theories.

Methodology: The study used the desk top library research approach to survey relevant extant literatures on investors' behaviour in relation to cumulative prospect theory and Radner theory in a comparative manner.

Results: Findings indicate that investors' behaviour in investment/consumption decision making is predicated on attitude to risk/uncertainty. They prefer higher return to lower risk; higher “satisfaction from commitment of wealth to asset bundle under condition of general equilibrium. These behavioural dispositions have been observed and addressed in the cumulative prospect theory and Radner theory.
Radner theory. The finding of this study is that the cumulative prospect and Radner theories serve as the barometers with which investors’ direction of investments are constantly monitored in the stock market globally.

**Recommendation:** This study therefore recommends that financial analysts and market participants should frequently combine the rudiments of the traditional finance and behavioural finance in analyzing investments as well as observing reactions of myriad competing investors, particularly in perfect markets or in incomplete markets.

*Keywords:* Investors’ behavior; cumulative prospect theory; radner theory and general equilibrium.

1. **INTRODUCTION**

A comparison of the cumulative prospect theory and Radner theory in economics and finance researches often begins with the understanding of utility theory, expected utility theory and prospect theory. Each of these theories deals with expected outcome from a gamble/ game. Each of the theories relates attitudes of investors towards risk and uncertainty. Naturally people always make choice under uncertain on a daily basis. For example, individuals most often make certain choices under conditions of uncertainty on daily basis at home, in marriages, religious activities, work places, taking studies in schools and even in gambling or lottery, just but to list a few.

Similarly, pursuing a degree in courses such as accounting, economics, banking and finance, pharmacy, medicine, and among others may lead to a lucrative employments or to unemployment. In the sciences, a medical doctor's appointment may result in the early detection and treatment of a disease, or it may be a waste of money. All these decisions portray varying attitude to risk under uncertainty in real life. This is so because life itself is full of uncertainty and risks. Business life, marital life, education life, political race/ ambition, just but to list a few is full of uncertainty and myriad risks with differing probability weighting assigns to it by rational individuals/investors.

Investors are known to commonly maintain three types of behaviours, ranging from risk seeking (risk loving), risk –averse and risk neutral. The rational agents normally attach probability weighting over expected future outcomes under uncertainty. This presupposes that investors are rational agents and vary in their attitude towards risk and uncertainty. They preferred to avoid losses to have investment gain /value. The utility or gain derivable from an expected outcome is a primary concern to investors. Utility therefore offers a rational method of expressing optimization behaviour with varying attitudes to risk in a situation of uncertainty.

2. **LITERATURE REVIEW**

2.1 **Utility Theory**

In micro economics, utility is analyzed under theory of utility. Utility is the satisfaction an agent gets from the consumption of a commodity. The utility theory tries to explain the rationality of individuals in decision making under risk and uncertainty. Two approaches, ordinal and cardinal, are used to analyze the utility of a consumer or investor. The weakness of each of these approaches of utility theory led to the development of the expected utility theory.

2.2 **Expected Utility Theory**

The expected utility theory explains that individual decisions over risky outcomes often deviate from the predicted outcomes or results. The original idea of expected utility theory was first proposed by Daniel Bernoulli in 1738. Bernoulli claimed that utility can be understood as the measurement of individual's satisfaction from consumption of his wealth. Von Neuman and Mongentsern [1] later popularized the expected utility theory through the use of a set of axioms which include transitivity and continuity on preference. They later made the expected theory a very powerful instrument at explaining the decision making of individuals or investors. The tenet of the expected utility theory is that investors or individuals are uniformly risk averse. However, indications abound where human behaviour may significantly contradicts this expected utility theory tenet particularly when facing uncertainty.

Expected utility theory is both a *normative* and *descriptive* theory of how people make decisions. It is a theory which explains while it may not be accurate to model the psychological mechanisms of decision-makers, or correctly predicts people's
choices. The obvious drawback of the expected utility theory led Kahneman and Tversky [2] to propose the prospect theory.

2.3 Prospect Theory

The prospect theory was first advanced by Markowitz (1952) under mean-variance approach to determining assets final outcome in the financial markets. The theory was later popularized by Kahneman and Tversky [2] where they explained that people value gains and losses differently. According to the prospect theory, people often base their decisions on perceived gains instead on perceived losses. Gains matter a lot to rational investors compared to losses they may suffer from an investment final outcome [3]. The theory also implies that people evaluate these losses and gains using certain past experiences. Though the prospect theory managed to explain some major violation of the expected utility theory which relates to small number of outcomes, its weakness is that it clearly violate the first order stochastic dominance. Similarly, it was incompatible with large number of outcomes. Also the prospect theory was unable to explain the source of uncertainty peculiar to individual decision making.

2.4 Cumulative Prospect Theory

As a way of ensuring that the noticeable lacuna in prospect theory was addressed, Tversky and Kahneman in [4] developed the cumulative prospect theory (CPT). Two central assumptions in CPT are that individuals are risk-averse over gains and risk-seeking over losses. They opine that rational agents tend to overweight low probability events while underweighting the likelihood of high-probability ones. Cumulative prospect theory (CPT) is based on the earlier version of prospect theory where it still characterizes decision makers deciding on the basis of the value function exhibiting loss aversion, diminishing sensitivity, and the probability distortion function. The stands of the cumulative prospect theory (CPT) of Tversky and Kahneman of [4] on one hand lie squarely on loss aversion. Generally, individual investors are both risk seekers and risk averters at the same time and this is frequently observed in their investment behaviour.

According to Hasbauch, Krause and Vesterlund (2007) were intellectually triggered to examine the robustness fourfold pattern of risk attitudes under two elicitation procedures. According to them, individuals are on average, risk-seeking over low-probability gains and high-probability losses and risk-averse over high-probability gains and low-probability losses with regard to prices for the gambles. Hasbauch, Krause and Vesterlund (2009) inquisitive assessment of the fourfold pattern is a very good predictor of risk attitudes but it appears to work only or better when people are asked to report their willingness to pay for a risky prospect. When they are instead asked to choose between two gambles (lotteries) and given expected value, it is often obvious that their decisions are not distinguishable from random choice.

2.5 Elements of Cumulative Prospect Theory

The main issues in the cumulative prospect theory is that cumulative probabilities rather than the probabilities are transformed are commonly. Consequently, extreme events which occur with small probability rather all small probability events are over weighted. He and Zhou [5] assert that in the context of financial asset allocation, the key elements of cumulative prospect theory (CPT) are that people evaluate assets in comparison with certain benchmarks, rather than on final wealth positions; people behave differently on gains and on losses; they are not uniformly risk averse and are distinctively more sensitive to losses than to gains (the latter is a behaviour called loss aversion).

2.6 Radner Theory

The Radner theory is otherwise referred to as Radner Equilibrium (RE). Roy Radner (1967) and (1968) reconsidered the Arrow-Debreu model and introduced the difference in information across agents, but does not come to grips with market incompleteness. Arrow [6] and Debreu [7] work shows how the analysis could be extended to a general setting with time and uncertainty. Radner constructed a coherent model of rational expectation where trading and uncertainty became explicit. This however differs from traditional complete markets general equilibrium model, where value (utility) and value (utility) alone decides budget feasibility.

The decisive contribution of Radner theory comes with "existence of equilibrium of plans, prices and price expectations in a sequence of markets" which appeared in 1972. Radner had to later generalize the theory in 1979. The 1972
paper considered a model with several goods, several time periods and general technologies. Agents are endowed with single-value and common correct expectations about future prices. For instance, the aspect of collective decisions in firms is eschewed by endowing firms with preferences of their own. Radner (1972) proved the existence of allocation, at which the trades and plans of all agents are optimal, and all markets clear, provided share prices on the stock market are uniformly positive, a gap that has been partly filled in literature when more suitable decision criteria for firms were introduced.

Radner (1968) concluded that if economic decision makers have unlimited computational capacity for choice among strategies; even if there is uncertainty about the environment, and different agents have different information and different beliefs about the environment, they can apply the standard theorems on the existence and optimal competitive equilibrium. In Radner theory, there is no role for money and liquidity. All contracts are negotiated at the beginning of the history of the economy, and from then, on all actions are determined by selected strategies. Such strategies may of course take account of new information as it becomes available.

On the other hand, Radner (1968) argues that a demand for liquidity arises from computational limitations and would be present even in a world of certainty about the environment if that world were sufficiently complicated. Radner (1968) suggests further that there is a basic difficulty in incorporating computational limitations in a "classical" equilibrium theory based on optimizing behavior, and that this presents an obstacle to an extension of the classical theorems of welfare economics to cover the case of a monetary economy.

In the same vein, Radner in 1968 also argue that if decision makers receive information about each other’s behaviour as well as about the environment, then this introduces a type of externality (interdependence) among their decision rules. This type of externality usually ensures that decision makers must take account of uncertainty about each other’s behaviour as well as about the environment. It may give rise to a demand for liquidity. In particular, the introduction of “spot” markets as well as futures markets results in this type of externality, and thus does not appear to be amenable to analysis by means of the “classical” theory of competitive equilibrium.

2.7 Arrow-Debreu-McKenzie (A-D-M) Model

The Arrow – Debreu model, also called the Arrow – Debreu – Mckenzie model points out that with perfect competition and demand independence assumptions, a set of prices exist such that aggregate supplies equal aggregate demands for every commodity in the economy. The general equilibrium model named after Kenneth Arrow (1964) and Gerard Debreu [7] work has become important in financial economics. The significance of Arrow – Debreu economy is in reference to arrow – Debreu security.

Arrow – Debreu securities is a security that pays one unit of a financial return if a particular state of the world is reached and nothing (zero) in other states. Arrow – Debreu model of state security promised future payments in terms of both the times at which payments are to be made and the states of the world that must obtain for payment to be made, hence it is a time – state paradigm sort of. For instance, a decision being made today that will generate outcome say go days from today (2 time period) and the outcome could be positive or negative (two possible future states of the world with associated payoffs). Thus, the states are mutually exclusive, that is only one can occur at the point in the future and exhaustive, one must occur.

So, Arrow – Debreu security characterizes uncertainty in investment outcome uncertainty refers to an incomplete state of knowledge about the truth of a proposition, particularly about which among many possible outcomes may result from a certain course of action. An uncertainty is more often described by probability. In areas in finance and related fields of study, uncertainty is interchangeably used as risk and risk is the variability of the outcomes of uncertain event. Thus, to investigate investors behaviour in terms of investment decision under uncertainty requires a clear understanding of attitude towards risk and utility theory.

Herves-Beloso, Moreno-Garcia, and Yannelis (nd) assert that the classical Arrow-Debreu-McKenzie (A-D-M) model consists of a finite number of commodities and a finite number of agents, each of whom is characterized by her preferences and initial endowments. Arrow [6] and Debreu [7] introduced uncertainty into the classical A-D-M model and showed how the deterministic results are still valid in the presence of uncertainty.
Radner (1968) introduced differential (asymmetric) information into the A-D-M model. In particular, an exchange economy with differential information consists of a finite set of agents, each of whom is characterized by a random utility function, a random initial endowment and a private information set. For such an economy, Radner (1968) defined a notion of Walrasian equilibrium, called Radner equilibrium. This notion is analogous to the Walrasian equilibrium in the arrow-Debreu-McKenzie deterministic model.

### 2.8 The Walrasian Equilibrium

The Walrasian equilibrium notion for an economy with differential information (Radner equilibrium) is of interest because it captures trades under asymmetric information. In such an economy, agents maximize ex ante expected utility subject to their budget constraint. However, each agent’s allocation is measurable with respect to its own private information and thus, all choices made reflect the informational asymmetries. It should be noted that the Radner equilibrium differs from the rational expectations equilibrium (REE), Radner (1979), which is an interim concept allowing prices to reveal some or all of the private information in the economy.

A major criticism of the RE is that it does not provide an adequate explanation as to how prices reveal the same information to agents who are differentially informed and therefore prices do not reflect the differential information of agents. This is not an issue for the Radner equilibrium since decisions are made in an ex-ante stage. However, since net trades are private information measurable for each agent, the equilibrium outcome reflects the asymmetric information.

### 2.9 Loss Aversion

Loss aversion in finance may also be regarded as break-even or disposition effect. It is defined as a reluctance to sell investment after they have fallen in value. In other words, loss aversion is the tendency for investors to hold on to losing stocks for too long and sell winning stocks too soon [8]. The most logical course of action would be to hold on to winning stocks in order to further gain and to sell losing stocks in order to prevent escalating losses. But the fact of the fact is that often time, many of the losing stocks never recover and the losses incurred continued to increase with often disastrous results [9].

Loss aversion also implies that decision making is sensitive to the description of the action choices, i.e to the way the alternative are framed (Kahneman & Tversky, 1990). A lot of persons decide differently depending on whether the problem is framed as contemplated gains or as losses. This losses salience effect extends to the financial decision problem and implies that people care more about the financial losses than gains. The losses are amplified at the social level to the extent that conversation or media reporting are biased towards transmitting adverse and emotionally charged news (Heath, Bell & Sternberg, 2001).

Loss aversion within the context of cumulative prospect theory is very relevant today in finance because it helps to explain some illogical financial behavior of people. For example, some workers may not want to put their money in a savings of fixed deposit accounts in the bank with a view to earning interest or they may refuse to work overtime because they don’t want to pay more taxes. Even though these workers will have financial benefit from the additional tax income, CPT points out that the benefits or utility gained from the extra money is not enough to overcome the feeling of loss incurred by paying taxes.

### 2.10 General Equilibrium

In the view of Stigler (2005), the theory of general equilibrium is the theory of interrelationship among all parts of the economy. General equilibrium exists when all prices are in equilibrium; each consumer spends his given income in a manner that yields him the maximum satisfaction; all firms in each industry are in equilibrium at all prices and output; and the supply and demand for productive resources (factors of production) are equal at equilibrium prices. There are certain assumptions that must be held sway for the general equilibrium to occur.

The general equilibrium analysis is based on the assumptions that there is perfect competition both in the commodity and factor markets; tastes and habits of consumers are given and constant; incomes of consumers are given and constant; factors of production are perfectly mobile between different occupations and places amongst others. Given these assumptions, the economy is in a state of general equilibrium when the demand for every commodity and service is equal to the supply for it. It implies perfect harmony of the decisions made by all the market participants.
The decisions of consumers for the purchase of each commodity must be in perfect accord with the decisions of producers for the production and sale of each commodity. Similarly, the decisions of owners for selling each factor service must be in perfect harmony with the decisions of their employers. It is only when the decisions of buyers of goods and services fit in perfectly with the decisions of sellers that the market is in general equilibrium.

2.10 Implications of Cumulative Prospect Theory on Financial Market

Cumulative prospect theory has been applied to diverse range of situations which before now appears to be inconsistent with the standard economic rationality. More specifically, it has been applied in the area of equity premium puzzle by Holdsworth and Mare (2014). Mehra and Prescott (1985) in the US stock exchange and South African stock market; the asset allocation puzzle by Li, Subrahmanyam and Yang (2014) on the Hang Seng Index; and Kahneman and Tversky [2]; the status quo bias by Rieger and Wang [10]; various gambling and betting puzzles by Von Neumann and Morgenstern [1], and Wu (1994); and intertemporal consumption and the endowment effect respectively.

2.11 Implications of Radner Theory on Financial Market

- It is the basis of investors’ investments in securities in the market in that they are always uncertain about returns.
- Radner equilibrium is an action framework for competition in the financial market
- It assists to determine equilibrium between market participants and investors in the financial markets.
- It tells us why investors’ utility is made due to budget feasibility.
- The Radner equilibrium engenders demand for liquidity in the financial markets as a result of computational limitation of investors.
- The Radner theory is of interest to agents in the economy or financial market in that it captures trades under information asymmetry. In such an economy, agents maximize ex ante expected utility subject to their budget constraint. However, each agent’s allocation is measurable with respect private to its own private information and thus all

2.12 Similarities between Cumulative Prospect Theory and Radner Theory

Comparatively, there is similarity between the cumulative prospect theory and the Radner theory/ equilibrium.

- First, they both address investors’ behaviour with regard to uncertainty in the markets.
- They try to spell out the utility value (gain) derivable from investment/commodity occasion by budget feasibility or initial endowment.
- Similarly, they are both coherent models of rational expectation where trading and uncertainty become explicit.
- Both the cumulative prospect theory and Radner theory clearly explain investors (individuals and firms) attitude to risks; given that firms are sometimes risk neutral and workers are risk averse.
- Given that the cumulative prospect theory and Radner theory are applicable particularly in an environment of uncertainty, they serve as a guide/compass at directing or cautioning managers at investing resources in order to maximize shareholders wealth.
- Cumulative prospect theory and Radner theory share partly from the traditional finance and of course from behavioural finance in that they are partly models for explaining sometimes the rationality and irrationality of an agent or investors.

2.13 Differences between Cumulative Prospect Theory and Radner Theory in Financial Markets

- Radner equilibrium differ from the rational expectation theory (utility theory, expected utility theory, prospect theory and cumulative theory) in that it allows price to reveal some or all of the private information in the economy.
- Cumulative prospect theory compares actual values with expected values from a gamble or lottery by probability weighting and with this, investors are able to maintain different attitude to risks in an environment of uncertainty.
2.14 Drawbacks of Cumulative Prospect Theory and Radner Theory

Though they both attempt to explain the rationality of the investors in their behaviour but do not explicitly explain how information come to impound on stock prices, neither do they account for price formation/ discovery, transparency in the financial markets as commonly observed and discussed in market microstructure. Similarly, cumulative prospect theory and Radner equilibrium do not tell us how volatility are transmitted into an economy or financial markets, only that they are concern about investors rationality and fear of uncertainty.

3. CONCLUSION AND RECOMMENDATIONS

There are several indications that the theory of incomplete markets provides a useful framework for unifying and clarifying the mutual dependence between real, financial and monetary phenomena. The contributions of Roy Radner (1972) were indeed seminal to the emergence of several economics and finance theories that keeps evolving. One of the assumptions of the capital market theory as advocated by Fama (1970) is market efficiency; and that investors in capital market are supposed to act according to the rationalism provided by the financial theories. However, empirical evidence has suggested that it is not so. Behavioural finance has provided new models to understanding the behaviours and functioning of the market participants who are not rational.

Findings from many economists and financial experts the world over have done a good number of interesting studies in the area influenced by cognitive psychology and economic behaviour in behavioural finance and investor’s behaviour. The realities and conclusions put forward by prospect theory and cumulative prospect theory is aversion, disposition effect, reference point, mental accounting and heuristic biases play important roles in shaping the cognitive behaviour of investors in the capital market rather than the rationalism provided by the financial market theories. Therefore, in the context of the Nigerian capital market, cumulative prospect theory should be applied in the area of asset allocation, bonds and equity investment, equity premium puzzle, either focusing on their performance or awareness and perception of bonds and equity investors from the behavioural perspective.

3.1 Suggestions for Further Studies

- The authors suggest that empirical studies be conducted by future researchers to examine investment behavior of rational agents in developed and developing financial markets with special reference to tenets of the cumulative prospect theory (CPT) using structure questionnaire, secondary data set and carefully analyzed with the aide of both parametric and non-parametric estimation methods.
- There is need for further researches to investigate some psychological/ emotional factors such as emotional intelligence, feelings, personality, news of loss, biases and crowd influences as drivers of investment behavior among rational agents in the financial markets in varying jurisdiction and climes.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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